Chapter Thirteen.

Standard scores and the normal distribution.

Situation

Test 1 27	Kym sits a Mathematics test and achieves a mark of 27. In the next test she scores 30. Has she improved?	Test 2 30

Before answering this question we might first ask:
What was each test out of?

$\frac{27}{40}$	Suppose that test 1 was out of 40 and test 2 was out of 50. Can we now decide whether she has improved?	$\frac{30}{50}$

Before answering we may want to know if the tests were of similar difficulty. What was the mean mark in each test?

Mean 23	Suppose the mean in test 1 was 23 and in test 2 was 25. Now can we judge whether her test 2 mark shows an improvement?	Mean 25

What if we also knew the standard deviation for each test as well?

St dev 5	Suppose the standard deviation in test 1 was 5 marks and in test 2 was 10 marks.	St dev 10

Now can you suggest whether or not Kym's mark in test 2 was an improvement on her mark in test 1?

Standard scores.

In the *situation* on the previous page did you consider expressing Kym's test scores in terms of the number of standard deviations each was from the mean? (An idea also encountered in one question of Miscellaneous Exercise Three earlier in this book.)

Expressing a score as a number of standard deviations above or below the mean is called **standardising** the score. We obtain the **standard score.**

$$\text{Standardised score} = \frac{\text{Raw score - mean}}{\text{standard deviation}}$$

Example 1

Jennifer scores 23, 35 and 17 in tests A, B and C respectively. If the mean and standard deviation in each of these tests are as given below express each of Jennifer's test scores as standardised scores.

Test A:	mean	30	standard deviation	5
Test B:	mean	32	standard deviation	6
Test C:	mean	15	standard deviation	2·5

In Test A Jennifer's standardised score is $\frac{23-30}{5}$ i.e. $-1{\cdot}4$.

In Test B Jennifer's standardised score is $\frac{35-32}{6}$ i.e. $0{\cdot}5$.

In Test C Jennifer's standardised score is $\frac{17-15}{2{\cdot}5}$ i.e. $0{\cdot}8$.

Exercise 13A.

1. Express each of the following as a standard score.
   (a) A score of 65 in a test that had a mean of 60 and a standard deviation of 5.
   (b) A score of 72 in a test that had a mean of 55 and a standard deviation of 10.
   (c) A score of 50 in a test that had a mean of 58 and a standard deviation of 4.
   (d) A score of 60 in a test that had a mean of 58 and a standard deviation of 4.
   (e) A score of 58 in a test that had a mean of 64 and a standard deviation of 8.

2. SuMin scores 30, 50, 7 and 26 in tests A, B, C and D respectively. If the mean and standard deviation in each of these tests are as given below express each of SuMin's test scores as standardised scores.

Test A:	mean	20	standard deviation	4
Test B:	mean	60	standard deviation	10
Test C:	mean	6	standard deviation	0·8
Test D:	mean	25	standard deviation	5

3. All of the first year students on a particular technology course sat exams in the core subjects of Mathematics, Chemistry, Electronics and Computing. The exam results produced the following summary statistics:

Mathematics exam:	mean mark	60	standard deviation	10·4
Chemistry exam:	mean mark	72	standard deviation	7·2
Electronics exam:	mean mark	48	standard deviation	14·6
Computing exam:	mean mark	63	standard deviation	7·4

One students scored 56 in Mathematics, 74 in Chemistry, 39 in Electronics and 72 in Computing. Standardise each of these scores and rank the subjects for this student listing them from best to worst on the basis of these standard scores.

4. All year ten students in a particular region sat exams in Mathematics, English, Science and Social Studies. The exam results in these subjects produced the following means and standard deviations.

Mathematics:	Mean:	63	Standard deviation:	14
English	Mean:	64	Standard deviation:	10
Science:	Mean:	72	Standard deviation:	8
Social Studies:	Mean:	106	Standard deviation:	22

One student achieved the following scores:
76 in Mathematics, 75 in English,
78 in Science, 104 in Social Studies.
Rank the four subjects in order for this student, highest standardised score first.

5. Jill and her boyfriend Jack sit the same maths exam, along with the 156 other candidates studying the course for which the exam formed a part of the assessment.

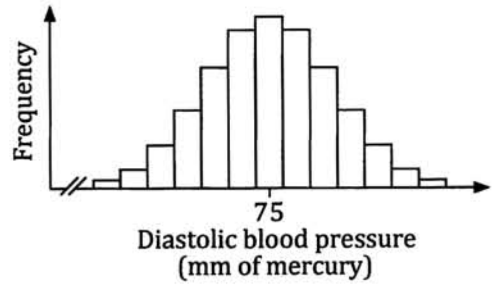
- ☛ The exam was marked out of 120.
- ☛ The mean mark for the entire 158 students was 65·2 and the standard deviation of the marks was 8·8.
- ☛ Jill scored 74 out of 120 and Jack scored 63 out of 120.

Complete the three incomplete responses from Jill shown below in the following conversation between her and her mother:

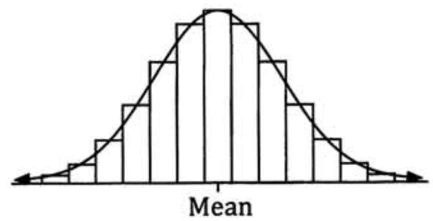
Jill (arriving home from school): *"Hi Mum. How's your day been?"*
Jill's mum: *"Pretty good dear. How was yours? Did you get any marks back from the exams you did."*
Jill: *"Yeah I got my maths mark."*
Jill's mum: *"What did you get?"*
Jill, quoting her exam mark as a standard score replied:
"Well I got _____ ."
Jill's mum: *"What! That sounds awful! What was the average?"*
Jill, again quoting standard scores:
"The mean was _____ ."
Jill's mum: *"What! What did Jack get?"*
Jill: *"Oh he got _____ ."*
Jill's mum (who knew something about mathematics):
"Wait a minute. Are we talking standard scores here?"

Normal distribution.

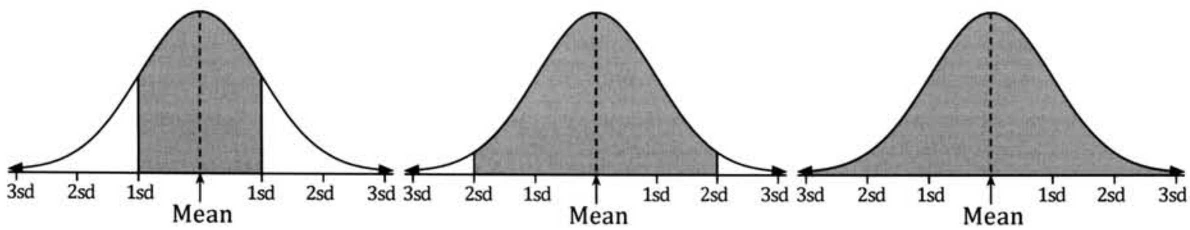
Suppose the diastolic blood pressure of a large number of adults was measured and the mean value was found to be 75 mm of mercury (mm of mercury being the units blood pressure is measured in). The data collected, if presented as a histogram, could well have a shape similar to the diagram shown on the right, i.e. a symmetrical distribution with many values close to the mean and the number of values decreasing as we move further from the mean.



Fitting a smooth curve to the midpoints of the columns we obtain a "**bell shaped curve**" as shown on the right.



If we make many measurements of something that occurs naturally, for example the heights of many adult females, the weights of many domestic cats, the foot lengths of many adult males, etc., the histogram of the data often follows this sort of shape.
Data of this kind is said to be **normally distributed**. In **normal distributions** approximately two thirds of the population lie within one standard deviation of the mean, 95% would lie within two standard deviations of the mean and almost all would lie within three standard deviation of the mean.
This is the 68%, 95%, 99·7% rule.



68·3% of area under curve shaded

95·4% of area under curve shaded

99·7% of area under curve shaded

In terms of probabilities, we could say that the probability of a randomly selected individual from a normally distributed population being within

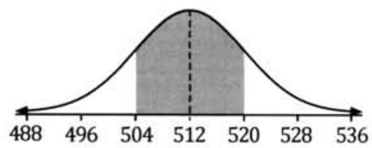
- one standard deviation of the mean is 0·683,
- two standard deviations of the mean is 0·954,
- three standard deviations of the mean is 0·997.

Note: The normal distribution is also referred to as the Gaussian distribution, after the German Mathematician Carl Gauss.

Example 2

A box of breakfast cereal has "contains 500 grams of breakfast cereal" printed on it. Suppose that in fact the weight of breakfast cereal contained in these boxes is normally distributed with a mean of 512 grams and a standard deviation of 8 grams. Determine the probability that a randomly chosen box of this cereal contains between 504 grams and 520 grams.

With a mean of 512 grams and a st dev^n of 8 grams:
504 grams is one standard deviation below the mean
and 520 grams is one standard deviation above the mean.



For normally distributed data the probability that a randomly chosen data point is within 1 standard deviation of the mean is, from the previous page, 0·683.
Thus the probability that a randomly chosen box of this cereal contains between 504 grams and 520 grams is 0·68.

The above example could be worked out using the "68" in the *68%, 95% 99·7% rule* because the question involved numbers of standard deviations that this rule relates to. What would we have done if instead the question had asked for the probability of a randomly chosen box of the cereal containing less that 500 grams? In this case 500 grams is 1·5 standard deviations below the mean, a situation not covered by the 68%, 95% 99·7% rule. In this case we can use the ability of various calculators to determine such probabilities, as the next example (which is again based on the breakfast cereal situation of example 1) shows.

Example 3

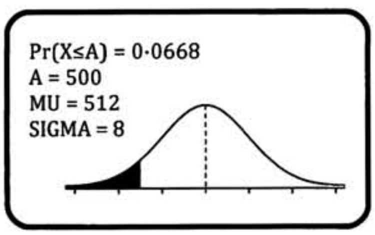
A box of breakfast cereal has "contains 500 grams of breakfast cereal" printed on it. Suppose that in fact the weight of breakfast cereal contained in these boxes is normally distributed with a mean of 512 grams and a standard deviation of 8 grams.

(a) Determine the probability that a randomly chosen box of this cereal contains less than 500 grams.

(b) In a random sample of 100 boxes of this cereal approximately how many boxes should we would expect to contain less than 500 g?

(a) For a randomly distributed set of values, with mean 512 and standard deviation 8, we require P(Randomly chosen value < 500).

Many calculators can display such information for normally distributed data.



The required probability is 0·0668.

The probability that a randomly chosen box of this cereal contains less than 500 grams is 0·0668.

(b) In any batch of boxes of this cereal we should expect that the proportion of them that contain less than 500 grams is about 0·07. Thus in a random sample of 100 boxes of this cereal we would expect approximately 7 boxes to contain less than 500 g.

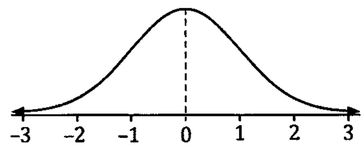
Using a calculator.

The various calculators have different capabilities and routines with regard to displaying probabilities for normally distributed sets of data.
You will gain familiarity with the ability of *your* calculator in this regard in the next exercise.

In the old days: Using a book of tables.

Prior to the ready availability of calculators with built in statistical routines for determining probabilities associated with normal distributions, these probabilities were determined using books of statistical tables.

These books give probabilities for just one normal distribution, the **standard normal distribution**. For this the random variable has a mean of 0 and a standard deviation of 1, as shown on the right.



Normal distributions having means and standard deviation not equal to these standard values needed to be standardised. We encountered this idea of standardising data by expressing it as a number of standard deviations above or below the mean at the beginning of this chapter. Calling the original score an "x score" and the standardised score a "z score" we have:

$$z \text{ score} = \frac{x \text{ score} - \text{mean of } x \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

Thus before the ready availability of sophisticated calculators, to answer the previous example which required us to determine the probability that from a normally distributed set of data, X, with mean 512 and standard deviation 8, a randomly selected item would have a value less than 500 we would have changed the 500 to a standard score:

$$\text{standard score} = \frac{500 - 512}{8}$$
$$= -1{\cdot}5$$

(i.e. a score of 500 is 1·5 standard deviations below the mean)

and then used the table of probabilities for the standard normal distribution to determine the required probability.

$$P(X < 500) = P(Z < -1{\cdot}5)$$
$$= 0{\cdot}0668$$

z	0·00	0·01	0·02	0·03
−1.9	0·0287	0·0281	0·0274	0·0268
−1.8	0·0359	0·0351	0·0344	0·0336
−1.7	0·0446	0·0436	0·0427	0·0418
−1.6	0·0548	0·0537	0·0526	0·0516
−1.5	0·0668	0·0655	0·0643	0·0630
−1.4	0·0808	0·0793	0·0778	0·0764
−1.3	0·0968	0·0951	0·0934	0·0918

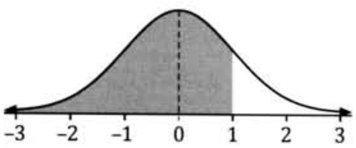
Thus, as before, the probability that a randomly chosen box of the cereal contains less than 500 grams is 0·0668.

Exercise 13B

Get to know the capabilities of your calculator with regard to normal probability distributions.

The questions of this exercise refer to data sets involving normally distributed scores, *X*. Using your calculator make sure that you can obtain each of the probabilities given in questions 1 to 8 below (correct to 4 d.p.), and each value of k in questions 9 to 17.

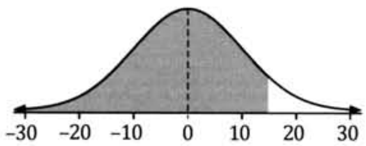
1. mean = 0
standard deviation = 1



$P(X < 1) = 0{\cdot}8413$

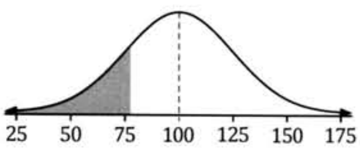
Can you also get 0·84 using the 68%, 95%, 99·7% rule?

2. mean = 0
standard deviation = 10



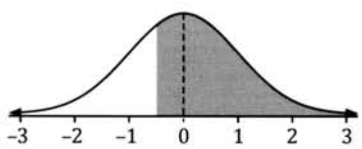
$P(X < 15) = 0{\cdot}9332$

3. mean = 100
standard deviation = 25



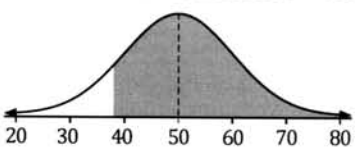
$P(X < 78) = 0{\cdot}1894$

4. mean = 0
standard deviation = 1



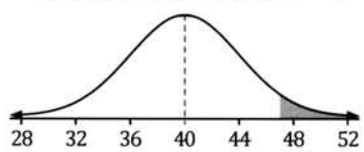
$P(X > -0{\cdot}5) = 0{\cdot}6915$

5. mean = 50
standard deviation = 10



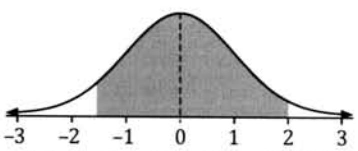
$P(X > 38) = 0{\cdot}8849$

6. mean = 40
standard deviation = 4



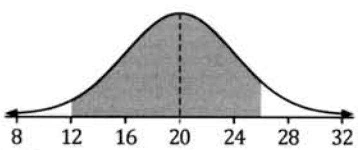
$P(X > 47) = 0{\cdot}0401$

7. mean = 0
standard deviation = 1



$P(-1{\cdot}5 < X < 2) = 0{\cdot}9104$

8. mean = 20
standard deviation = 4



$P(12 < X < 26) = 0{\cdot}9104$

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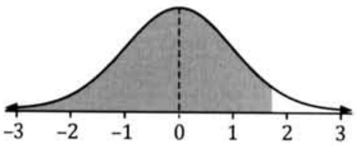
9.

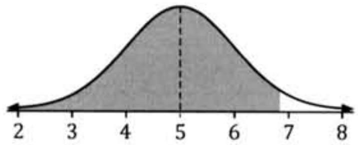
mean = 0
standard deviation = 1



$P(X < k) = 0{\cdot}9573$
$\therefore \quad k = 1{\cdot}72$ (2dp)

10.

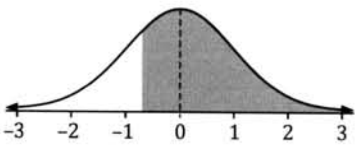
mean = 5
standard deviation = 1



$P(X < k) = 0{\cdot}9671$
$\therefore \quad k = 6{\cdot}84$ (2dp)

11.

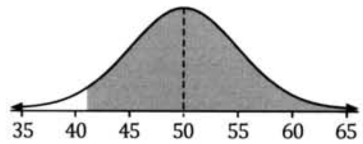
mean = 0
standard deviation = 1



$P(X > k) = 0{\cdot}7517$
$\therefore \quad k = -0{\cdot}68$ (2dp)

12.

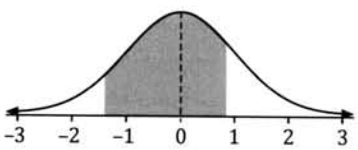
mean = 50
standard deviation = 5



$P(X > k) = 0{\cdot}9656$
$\therefore \quad k = 40{\cdot}9$ (1dp)

13.

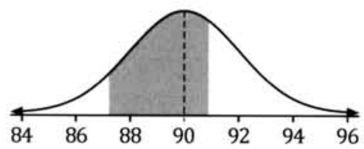
mean = 0
standard deviation = 1



$P(-1{\cdot}4 < X < k) = 0{\cdot}7215$
$\therefore \quad k = 0{\cdot}85$ (2dp)

14.

mean = 90
standard deviation = 2



$P(87{\cdot}2 < X < k) = 0{\cdot}5964$
$\therefore \quad k = 90{\cdot}92$ (2dp)

15.

mean = 40,
standard deviation = 15.
$P(X < k) = 0{\cdot}9850$
$\therefore \quad k = 72{\cdot}55$ (2dp)

16.

mean = 10,
standard deviation = 0·5.
$P(X > k) = 0{\cdot}0721$
$\therefore \quad k = 10{\cdot}73$ (2dp)

17.

mean = 0·1,
standard deviation = 0·01.
$P(0{\cdot}08 < X < k) = 0{\cdot}3036$
$\therefore \quad k = 0{\cdot}0955$ (4dp)

Notation

If we use X to represent the possible values of a normally distributed set of measurements having a mean μ and standard deviation σ (and hence variance σ^2) this is sometimes written:

$$X \sim N(\mu, \sigma^2).$$

"μ" is a Greek letter, mu, (pronounced myew) and σ is sigma so this is read as:

X is normally distributed with mean myew and standard deviation sigma.

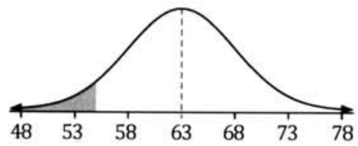
Example 4

If $X \sim N(63, 25)$ determine $P(X < 55)$.

X is normally distributed with a mean of 63 and a standard deviation of 5.

Using a calculator

$P(X < 55) = 0{\cdot}0548$



Using a tables book

$P(X < 55) = P(Z < -1{\cdot}6)$
$= 0{\cdot}0548$
(Shown for interest only.)

Example 5

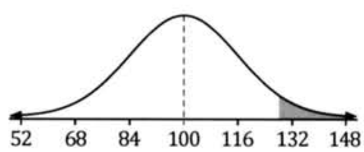
Eight thousand two hundred and forty students were given an IQ test. The scores were normally distributed with a mean of 100 and a standard deviation of 16.

(a) Determine how many of the students, to the nearest ten, achieved a score in excess of 128.

(b) What were the minimum and maximum scores of the middle 60% of students on this test?

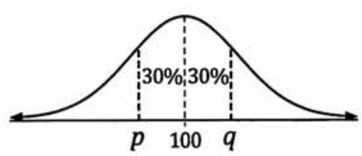
(a) Let X be the scores obtained in the test.
Thus $X \sim N(100, 16^2)$.
We require $P(X > 128)$.
Using a calculator, $P(X > 128) = 0{\cdot}0401$.
Number scoring more than 128:

$$0{\cdot}0401 \times 8\,240 \approx 330$$



Approximately 330 students achieved a score in excess of 128.

(b) If p is the lowest score achieved by the middle 60% then $P(X < p) = 0{\cdot}2$ i.e. $p = 86{\cdot}53$
and if q is the highest score achieved by the middle 60% then $P(X < q) = 0{\cdot}8$ i.e. $q = 113{\cdot}47$
(Some calculators can determine p and q more directly for this symmetrical situation.)

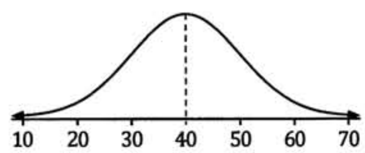


The lowest and highest scores achieved by the middle 60% are 86·5 and 113·5 respectively (to the nearest half mark).

Example 6

If $X \sim N(40, 10^2)$ determine each of the following probabilities using the 68%, 95%, 99·7% rule, and not the statistical capability of your calculator.

(a) $P(30 < X < 50)$ (b) $P(20 < X < 60)$ (c) $P(40 < X < 60)$ (d) $P(X \leq 50)$



(a) 30 is one standard deviation below the mean and 50 is one standard deviation above the mean.
Thus $P(30 < X < 50) = 0{\cdot}68$

(b) $P(20 < X < 60) = 0{\cdot}95$

(c) $P(40 < X < 60) = \dfrac{0{\cdot}95}{2}$
$= 0{\cdot}48$ (correct to 2 d.p.)

(d) $P(X \leq 50) = 0{\cdot}5 + \dfrac{0{\cdot}68}{2}$
$= 0{\cdot}84$

Note that we make no distinction between $P(X \leq 50)$ and $P(X < 50)$. Including the line or not makes no difference to the area of the region.

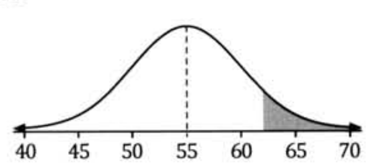
Example 7

Let us suppose that the time from Simon getting out of bed until his arrival at school is normally distributed with a mean of 55 minutes and a standard deviation of 5 minutes. Simon's arrival at school is classified as being late if it occurs after 9.10 am.

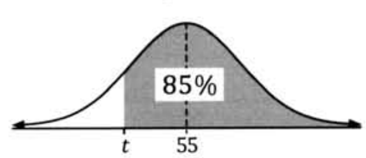
(a) One day Simon gets out of bed at 8.08 am. What is the probability of him arriving late?

(b) For a period of time Simon always gets out of bed at the same time but finds that he arrives late approximately 85% of the time! What time is he getting out of bed (to the nearest minute)?

(a) Let T minutes be the time from getting out of bed until arrival at school.
Thus $T \sim N(55, 5^2)$.
Simon has 62 minutes to get to school before he is late.
We require: $P(T > 62)$
Calculator gives: $P(T > 62) = 0{\cdot}0808$.
If Simon gets out of bed at 8.08 am the probability of him arriving late is 0·0808.



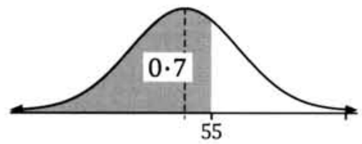
(b) The time that Simon is allowing himself to get to school is causing him to be late approximately 85% of the time.
We require t for which $P(T > t) = 0{\cdot}85$.
Calculator gives: $t \approx 49{\cdot}8$
Thus Simon is allowing approximately 50 minutes to get to school and for 85% of the days the journey takes longer than this, causing him to be late 85% of the time.
Simon is getting out of bed at 8.20 am.



Quantiles.

Quantiles are the values which a particular proportion of the distribution falls below.

Thus if 0·7 (70%) of the distribution is below 55 then 55 is the 0·7 quantile.



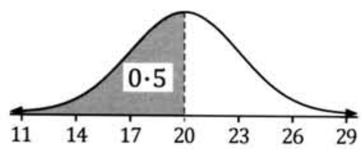
Alternatively we can refer to 55 as being the 70th **percentile**.

Note
- We are already accustomed to referring to the 0·25 quantile as the first, or lower, **quartile** and the 0·75 quantile as the third, or upper, quartile.
- If the quartiles divide a distribution in to four equal parts and the percentiles divide the distribution into 100 equal parts what might deciles and quintiles do?

Example 8

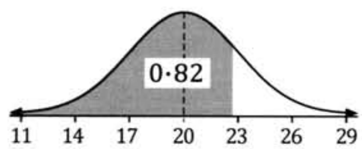
If $X \sim N(20, 3^2)$ determine (a) the 0·5 quantile, (b) the 0·82 quantile, (c) the 24^{th} percentile, (d) the 62^{nd} percentile.

(a)



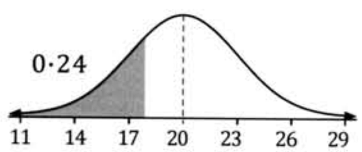
By inspection:
The 0·5 quantile is 20.

(c)



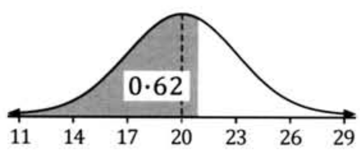
Using a calculator:
The 0·82 quantile is 22·7.

(c)



Using a calculator:
The 24^{th} percentile is 17·9.

(d)



Using a calculator:
The 62^{nd} percentile is 20·9.

Exercise 13C

1. The random variable, X, is normally distributed with a mean of 12 and a standard deviation of 2, i.e. $X \sim N(12, 2^2)$. Determine $P(X \geq 13{\cdot}5)$.

2. The random variable, X, is normally distributed with a mean of 240 and a variance of 400, i.e. $X \sim N(240, 20^2)$. Determine $P(218 < X < 255)$.

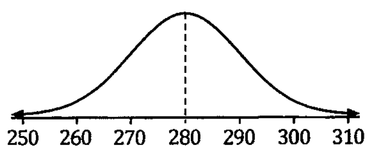
3. $X \sim N(62, 64)$, i.e. X, is normally distributed with a mean of 62 and a standard deviation of 8. Given that $P(X > k) = 0{\cdot}8238$ determine k.

4. If $X \sim N(0, 1)$ determine (a) the 0·72 quantile, (b) the 0·26 quantile, (c) the 89^{th} percentile, (d) the 23^{rd} percentile.

5. If $X \sim N(20, 3^2)$ determine
   (a) the 0·44 quantile,
   (b) the 0·74 quantile,
   (c) the 33rd percentile,
   (d) the 85th percentile.

6. Using the 68%, 95%, 99·7% rule, and <u>not</u> the statistical capability of your calculator, determine the following probabilities.
   (a) $P(-1 < X < 1)$, $X \sim N(0, 1^2)$.
   (b) $P(-2 < X < 2)$, $X \sim N(0, 1^2)$.
   (c) $P(-3 < X < 3)$, $X \sim N(0, 1^2)$.
   (d) $P(8 < X < 32)$, $X \sim N(20, 6^2)$.
   (e) $P(4 < X < 16)$, $X \sim N(10, 2^2)$.
   (f) $P(0 < X < 1)$, $X \sim N(0, 1^2)$.
   (g) $P(X < 1)$, $X \sim N(0, 1^2)$.
   (h) $P(X > 1)$, $X \sim N(0, 1^2)$.
   (i) $P(X < 5)$, $X \sim N(0, 5^2)$.
   (j) $P(X > 70)$, $X \sim N(60, 10^2)$.

7. Let us suppose that the duration of pregnancy, for a naturally delivered human baby, is a normally distributed variable with a mean of 280 days and a standard deviation of 10 days.



   Using the 68%, 95%, 99·7% rule, and <u>not</u> the statistical capability of your calculator, determine estimates for the following.
   (a) The percentage of human pregnancies, for naturally delivered babies, that are between 250 days and 310 days.
   (b) The percentage of human pregnancies, for naturally delivered babies, that exceed 290 days.
   (c) The percentage of human pregnancies, for naturally delivered babies, that are between 260 days and 270 days.

8. A machine produces components whose weights are normally distributed with a mean of 500 g and standard deviation of 5 g.
   (a) According to the 68%, 95%, 99·7% rule, what percentage of the components will have a weight of less than 495 g?
   (b) According to the 68%, 95%, 99·7% rule, what percentage of the components will have a weight of less than 490 g?

9. A box of breakfast cereal has "contains 300 grams of breakfast cereal" printed on it. Suppose that in fact the weight of breakfast cereal contained in these boxes is normally distributed with a mean of 310 grams and a standard deviation of 4 grams. Determine the probability that a randomly chosen box of this cereal contains
   (a) more than 312 grams of breakfast cereal,
   (b) less than 300 grams of breakfast cereal.

10. The lengths of adult male lizards of a particular species are thought to be normally distributed with a mean of 17·5 cm and a standard deviation of 2·5 cm.
Determine the probability that a randomly chosen adult male lizard of this species will have a length (a) less than 17·5 cm
(b) between 15 cm and 17·5 cm.

11. The scaled scores in a national mathematics test are normally distributed with a mean of 69 and a standard deviation of 12.
What is the probability that a randomly selected candidate who sat this test has a scaled score of
(a) more than 75 (b) between 66 and 75 (c) less than 45.

12. The heights of fully grown plants of a certain species are normally distributed with a mean of 30 cm and a standard deviation of 4 cm. If 100 fully grown plants of this species are randomly selected approximately how many would you expect to be:
(a) taller than 35 cm,
(b) shorter than 25 cm,
(c) between 25 cm and 30 cm in height.

13. Let us suppose that 44 mg is 110% of the recommended daily intake of a particular vitamin and that a 110 mL container of fruit juice contains approximately 44 mg of this vitamin. If in fact the weight of the vitamin in the 110 mL containers of the fruit juice is normally distributed with mean 44 mg and standard deviation 2·5 mg, determine the probability that a randomly chosen 110 mL container of this fruit juice contains less than the recommended daily intake of the vitamin.

14. Five thousand five hundred and forty two students sat a particular leaving exam. The scores obtained were normally distributed with a mean of 62 and a standard deviation of 12·5.
(a) Distinction certificates were awarded to students who gained a mark of 80 or more. How many students gained distinction certificates?
(b) A mark of less than 40 was regarded as a fail. How many of the students failed?

15. Let us suppose that the heights of the adults of a particular country are normally distributed with a mean of 1·75 m and a standard deviation of 10 cm. A car manufacturer wishes to design a new car with the space allowed for the driver, and the "travel" on the drivers seat, suitable for every adult in the population except the tallest 5% of the adult population and the shortest 5% of the adult population. What is the height of the shortest driver and the tallest driver that the manufacturer is attempting to allow for. (Answer to nearest half centimetre.)

16. The marks achieved in a particular exam are normally distributed with a mean of 64 and a standard deviation of 12.
Grades are to be awarded as follows:

Top 12%	of candidates:	Grade A
Next 25%	of candidates:	Grade B
Next 40%	of candidates:	Grade C
Next 15%	of candidates:	Grade D
Remainder	of candidates:	Grade F

Determine the marks that form the A/B, B/C, C/D, and D/F grade boundaries, giving your answers correct to the nearest whole number.

17. Let us suppose that the time, in minutes, from Monica leaving home until she arrives at work is a normally distributed random variable with a mean of 45 and a standard deviation of 5. Monica's arrival at work is classified as late if it occurs after 8.30 am.
   (a) One day Monica leaves home at 7.40 am. What is the probability of her arriving late?
   (b) For a period of time Monica leaves home at the same time each day. During this period she finds that she arrives late approximately 8% of the time. What time is she leaving home (to the nearest minute)?
   (c) What is the latest time (involving whole minutes) that Monica should leave home each day if she wishes to cut her late arrivals to less than 1%?

18. The annual rainfall in an area in the south west of Western Australia is normally distributed with a mean of 1 200 mm and a standard deviation of 200 mm.
According to this model, and assuming the situation does not change, in every one hundred years how many years would you expect the annual rainfall to be
   (a) less than 800 mm,
   (b) more than 1 500 mm,
   (c) between 800 mm and 1 500 mm.
   (d) (Challenge.) Given that a year has an annual rainfall of more than 1300 mm what is the probability that the rainfall for the year is less than 1500 mm?

19. The weight of each apple harvested from a particular orchard determines where the apple will be sent:

If weight of apple $\geq$ 250 g send to premium outlet
150g < weight of apple < 250 g send to normal market
weight of apple $\leq$ 150 g send for juicing.

The weights of the apples are normally distributed with mean 180 g and standard deviation 40 g.
   (a) In a random sample of 1000 apples how many would you expect to go to the premium outlet?
   (b) (Challenge.) Given that an apple does not go to the premium outlet what is the probability that it is sent for juicing?

Miscellaneous Exercise Thirteen.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary work section at the beginning of the book.

1. A particular straight line with a gradient of m and cutting the y-axis at the point with coordinates (0, c) has equation $y = mx + c$.
   (a) The line passes through the point (3, 4).
   Which of the following equations must be true?

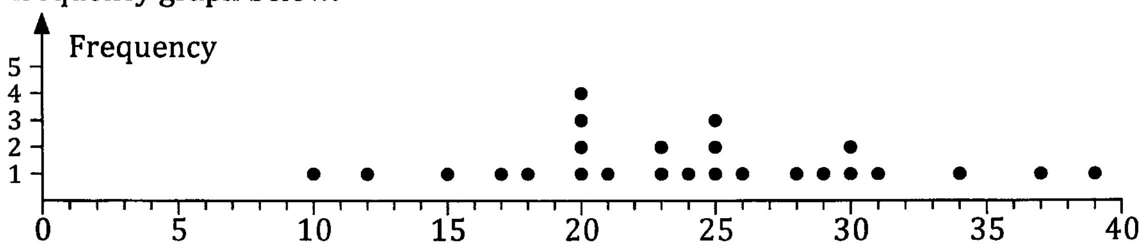
Equation 1	Equation 2	Equation 3
$y = 3x + 4$	$3 = 4m + c$	$4 = 3m + c$

   (b) The line also passes through the point (8, 19).
   Which of the following equations must be true?

Equation 4	Equation 5	Equation 6
$8 = 19m + c$	$19 = 8m + c$	$y = 8x + 19$

   (c) Solve your equations from parts (a) and (b) to determine the equation of the straight line.

2. Find the mean, median, mode and range for the set of scores shown in the dot frequency graph below.



3. One hot dog and three burgers cost \$13·30. Four hot dogs and four burgers cost \$25·20. What would be the cost of four hot dogs and six burgers? (Assume that the individual cost of a hot dog and of a burger remain unchanged throughout.)

4. A rectangle is of length x cm and width y cm.
   Four times the width exceeds one length by 4 cm.
   The perimeter of the rectangle is 42 cm.
   Find the values of x and y and hence determine the area of the rectangle.

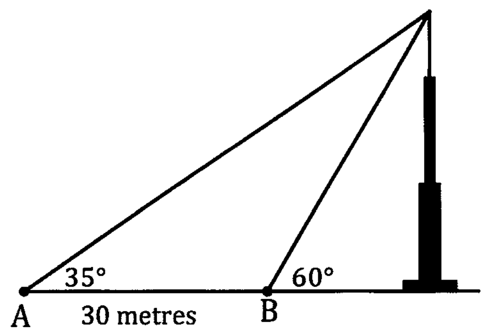
5. In support of a primary school fete a parent makes lots of high quality home made chocolates and lollies. The organisers decide to sell bags containing 20 chocolates for \$6·00 and bags containing 20 lollies for \$4·00. They also wish to sell a mixed bag of 20 chocolates and lollies for \$4·80. How many chocolates and how many lollies should each of these bags hold for the price of \$4·80 to be consistent with the prices of the other bags?

6. A road construction company charges 125 million dollars for constructing a 20 km stretch of highway and 245 million dollars for constructing 40 km of similar highway. Based on these costs and assuming a linear relationship exists between the total cost and the length of road constructed, determine the cost of constructing (a) 25 km of similar highway, (b) 52 km of similar highway.

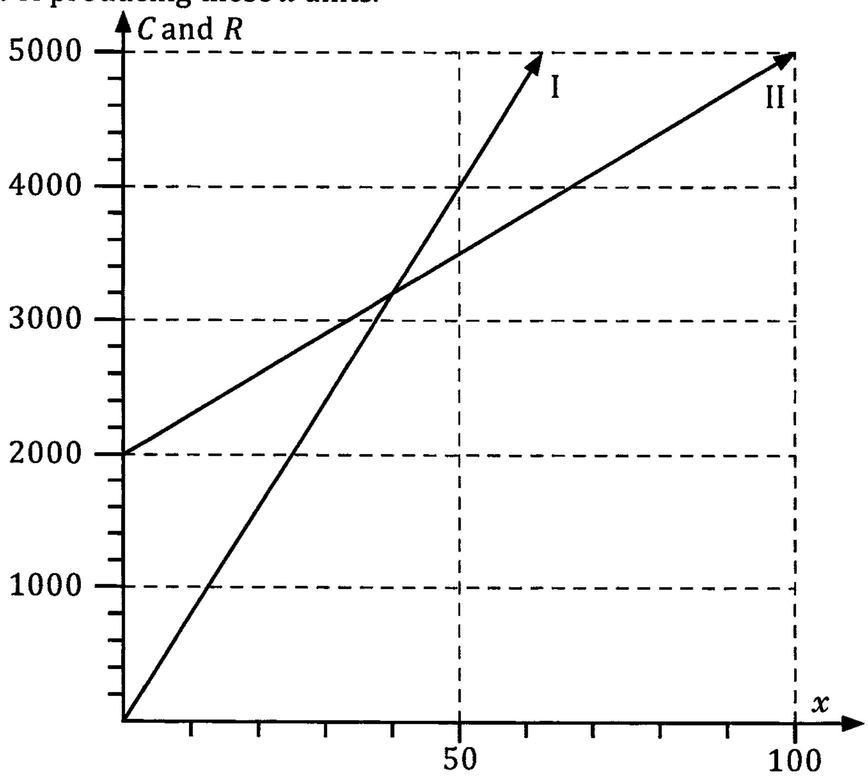
7. (a) Three consecutive integers have a sum of 504. Find the integers.
   (b) Three consecutive even integers have a sum of 504. Find the integers.

8. Jack walks 2·4 km on a bearing 060° followed by 4·4 km on a bearing 190°. On what bearing and for what distance should he now walk to return directly to his starting point?

9. From a point A, level with the base of a monument, the angle of elevation of the topmost point of the monument is 35°.
   From point B, also at ground level but 30 metres closer to the monument, the same point has an angle of elevation of 60°.
   Find how high the topmost point is above ground level. (Give your answer correct to the nearest metre.)

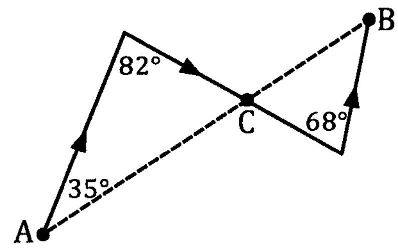


10. When a company sells x units of a particular product the revenue raised, $$R$, is given by one of the lines on the graph shown below and the other line shows $C, the cost of producing these x units.



   (a) Which of the two lines, I or II, is likely to be the revenue line and which the cost line? (Explain your answer.)
   (b) What does the graph suggest is the value of x for "break even" (i.e. revenue raised = cost of production.)
   (c) Suggest equations for each of the two lines shown.

11. A yacht travels from location A to location B by tacking, as shown in the diagram.
The direct distance from A to B is 580 metres.
The path the yacht takes causes it to cross the direct line from A to B at a point C where the distance from A to C is 65% of the distance from A to B.
How far did the yacht travel altogether in its journey from A to B?



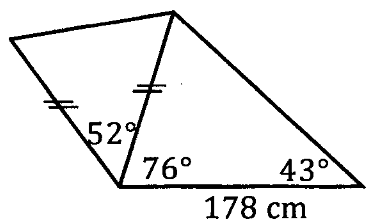
12. A real estate agent wants to quote an average house price for a particular region. He obtains the following information about the sale prices of the 29 most recent sales in the area:

Sale price	\$400 000 or less	\$400 001 → \$500 000	\$500 001 → \$600 000	\$600 001 → \$700 000
No. of sales	1	2	3	4

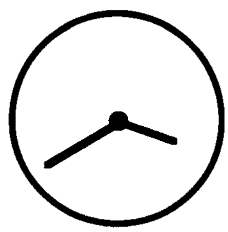
Sale price	\$700 001 → \$800 000	\$800 001 → \$900 000	\$900 001 → \$1 000 000	Over \$1 000 000
No. of sales	5	5	3	6

What should he quote as an average price? Justify your answer and include mention of any issues you consider relevant.

13. An ornate window is to consist of two triangular pieces of glass placed in an aluminium frame as shown in the diagram on the right.
Neglecting the thickness of the frame find
(a) the total length of the aluminium,
(b) the total area of glass.



14. Clearly showing your use of trigonometry determine to the nearest millimetre the distance between the tip of the 205 mm hour hand of a clock and the tip of the 312 mm minute hand of the clock at twenty minutes to four.



15. Twenty four students sat a test and the scores they obtained were as follows:

Initials	PA	CB	JB	CC	JD	KD	LF	LJ	MJ	EK	IM	PN
Score	35	19	47	25	39	30	9	34	41	33	39	29

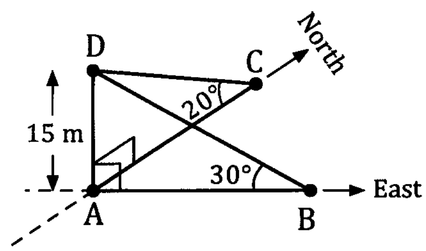
Initials	RN	PP	AR	TR	VR	AS	PS	TS	BV	PV	IW	RZ
Score	26	41	17	33	43	35	28	33	26	37	12	30

Grades of A, B, C, D and F are awarded according to the following rules:

		Score ≥	(mean + 1·5 × standard deviation):	Grade A
(mean + 0·5 × standard deviation)	≤	Score <	(mean + 1·5 × standard deviation):	Grade B
(mean − 0·5 × standard deviation)	≤	Score <	(mean + 0·5 × standard deviation):	Grade C
(mean − 1·5 × standard deviation)	≤	Score <	(mean − 0·5 × standard deviation):	Grade D
		Score <	(mean − 1·5 × standard deviation):	Grade F

Assign grades to each of the 24 students according to these rules.

16. Points A, B and C all lie on horizontal ground. A vertical tower DA has its base at A and is of height 15 metres. C lies due North of A and B is due East of A. The angle of elevation of D is 20° from C and 30° from B (see diagram).
Calculate (a) how far C is from A,
(b) how far B is from A,
(c) how far C is from B,
(d) the bearing of C from B.



17. The tank in a water irrigation system holds 80 000 litres of water.
The tank is initially three quarters full and each day, from 6am to noon, water flows from the tank at the rate of 1000 litres per hour.
This occurs for 6 days with no water entering the tank.
On day 7 rain is forecast so the system is switched off for days 7 and 8. This rain means that not only is no water taken off during these two days but instead, on day 7, from 6am to 6pm, 24 000 litres flows in. Unfortunately no water flows in on day 8.
The usual 6 am to noon daily outflow then recommences for days 9, 10, 11 and 12 and during these four days no more rain falls.
Sketch a piecewise graph showing the amount of water in the tank for these 12 days.

18. The 5347 candidates who sat a national mathematics test scored marks that had a mean of 127 and a standard deviation of 17.
Distinction certificates are to be awarded to the top 15% of students.
Participation certificates are to be awarded to students scoring less that 115.
(Candidates scoring between the above categories receive other certificates.)
The top 1% of students were awarded prizes.
Modelling the results as a normal distribution with mean 127 and standard deviation 17 determine each of the following:
(a) The lowest mark, rounded to the next whole mark *down*, that would achieve a distinction certificate.
(b) The number of students, to the nearest 10 students, who would receive a participation certificate.
(c) The lowest mark, rounded to the next whole number *up*, that would achieve a prize.
(d) The lowest and highest marks, each rounded to one decimal place, achieved by the middle 20% of students.

19. (Challenging) A river runs East to West. A tree stands on the edge of one bank and from a point C, on the opposite bank and due South of the tree, the angle of elevation of the top of the tree is 28°. From point D, 65 m due East of C the angle of elevation of the top of the tree is 20°.
Calculate (a) the height of the tree,
(b) the width of the river at C.